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The physics of the follow, the draw and the massé (in billiards and pool)

by Jearl Walker

Billiards and pool have the feel of physics. Balls collide with each other and the rails of the table like gas molecules in some two-dimensional tank. Actually the physics of billiards and pool is subtler. For example, a skilled player can impart spin to the ball, achieving such effects as the follow, the draw and the massé. Indeed, the interaction of the cue and the ball may be the most challenging application of classical mechanics. To master the forces and trajectories of both billiards and pool one must play the game often and analytically. A helpful step toward that objective is an understanding of the physics of the games.

Recently Todd King of Temple City, Calif., sent me his analysis of some of the classic shots in pool. Until a few years ago almost the only study concerning the dynamics of billiards was in lecture notes by Arnold Sommerfeld, who is better known for his work on early quantum mechanics. Last year David F. Griffing of Miami University devoted a chapter to pool and billiards as part of his book *The Dynamics of Sports: Why That's the Way the Ball Bounces.* These three sources inform the following discussion of the physics of billiards and pool. After describing some simple relations I shall take up a few of the famous trick shots outlined in *Byrne's Treasury of Trick Shots in Pool and Billiards,* by Robert Byrne.

When the cue strikes the cue ball, both horizontal and rotational motions are imparted. For simplicity assume that the cue stick is horizontal and delivers only a horizontal force. Although the force can be applied anywhere on the ball's surface facing the player, the shock sets the ball in horizontal motion just as if the force were applied at the center of mass.

Now assume that the stroke by the cue is in the vertical plane passing through the ball's center of mass, namely on an imaginary vertical line running through the center of the face toward the player. The location of the blow along this line has no direct bearing on either the ball's initial velocity or its momentum (the product of mass and velocity). They are set by two other factors in the collision. One factor, over which the player has virtually no control, is the duration of the collision. The second factor, easily controlled by the player, is the force on the ball. A "hard" shot generates more velocity and momentum than a "soft" shot because the force in the collision is greater.

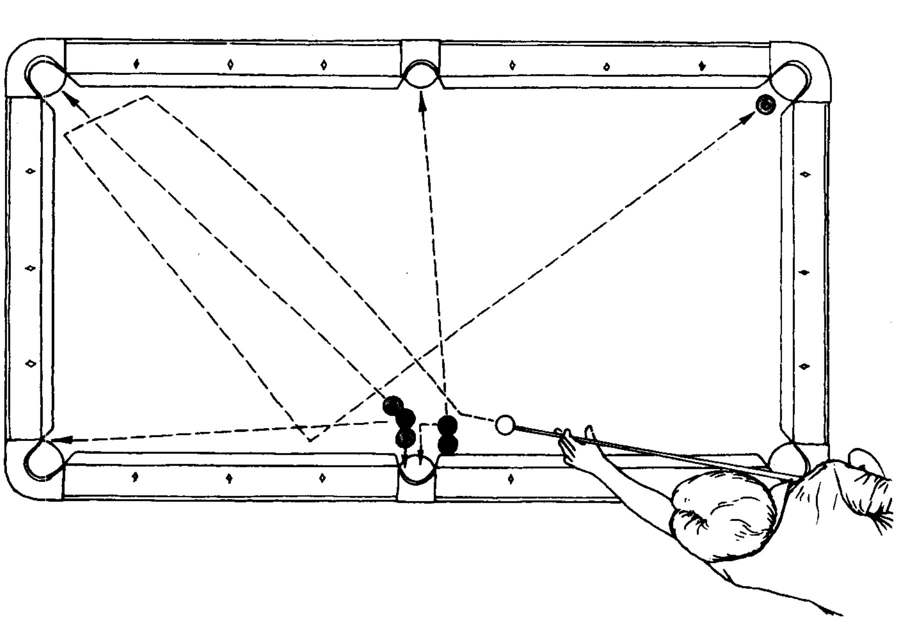
In addition to horizontal motion the cue also generates a torque that makes a ball rotate about its center of mass. The magnitude of the torque is equal to the product of the force and a lever arm that represents the vertical distance between the middle of the ball and the point where the cue strikes. The torque increases as the distance of the blow from the middle of the ball increases.

The torque determines the initial rate at which the ball spins about the center of mass. The spin is proportional to the torque divided by the ball's moment of inertia (a number that takes into account not only the mass of an object but also the distribution of the mass around the axis of rotation). For a cue ball rotating about an axis through its center of mass the moment of inertia is two-fifths of the mass multiplied by the square of the radius. The factor of 2/5, which arises from the shape of the ball, plays a role in a player's decision about where to stroke the ball in certain shots.

If the player wants an initially nonspinning ball, he should strike it at the height of its center of mass. The lever arm for such a stroke is zero, and so the torque and spin are zero. With a higher blow the collision has a lever arm and hence a measurable torque. The ball moves forward because of the force of the collision, and it spins about its center of mass because of the torque. The ball has topspin: the top of the ball moves away from the player faster than it otherwise would. Striking the ball below the center results in backspin.

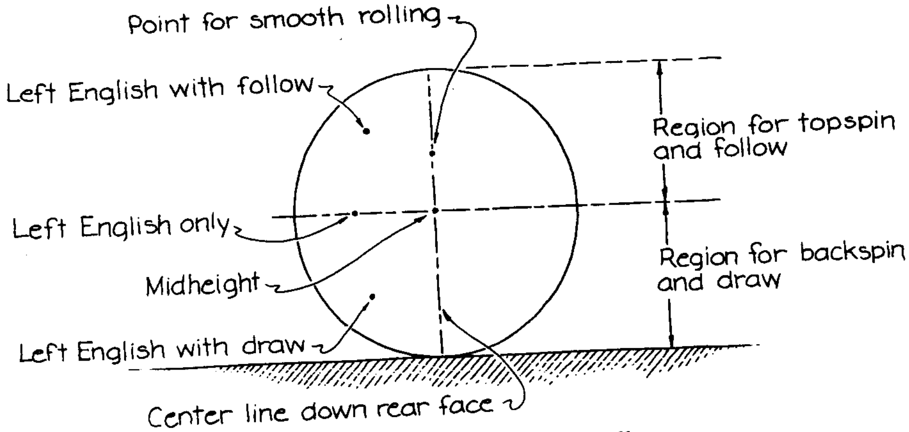
The player's stroke therefore controls three features of the motion. The force determines the velocity of the ball over the table. The lever arm of the force determines the direction of spin. The product of the force and the lever arm determines the rate of spin.

Without friction from the surface of the table the cue ball would continue moving until it hit a rail or another ball. Even a surface worn smooth from play can provide significant friction, however, if the ball slips on the cloth. The friction can be high enough to alter both the horizontal and the rotational motion of the ball and thereby significantly change the shot. If the ball rolls over the table without slipping, the friction is low and affects little more than the maximum distance of roll.



The "just showin' off" shot

The friction on a slipping ball depends on the weight of the ball and on the surface texture of the cloth and the ball, but it is independent of the rate at which the ball slips. The direction of the force depends entirely on the direction of slip. Suppose the player delivers a large topspin to the cue ball; the bottom surface then slides toward him and the center of mass moves away. At the point of contact the friction force is opposite to the sliding. (The force is away from the player.) Since the friction opposes the sliding, it begins to decrease the spin of the ball about its center of mass. And since the friction force is away from the player, it continues to propel the ball forward and away from the player. A ball given a large topspin can run for a long time because of this additional propulsion.



Strategic places to strike the cue ball

Suppose the player imparts backspin. The force sends the ball away and the torque sets it into a spin that makes the bottom slide over the cloth in the same direction. The friction force on the bottom on the ball is thus toward the player. Again the friction tends to slow the spin, but this time its effect on the center of mass is rearward. As the ball slides, its forward motion and spin are slowed. Eventually the backspin is eliminated and the ball begins to roll without sliding. A ball hit with backspin runs only a short distance because the friction opposes the motion of the center of mass.

A ball with topspin will slip unless the speed of its center of mass is equal to the spin multiplied by the radius. Then the ball's forward motion is exactly matched by the motion of the bottom surface through the point of contact with the table. To achieve this match instantly the player must stroke the ball at a point that is exactly above the middle by a distance equal to two-fifths of the radius. This relation is established by the fact that the 2/5 in the formula for the moment of inertia must be canceled by the 2/5 in the formula for the lever arm of the torque.

If the cue ball is struck higher than at this special point, the spin rate is at first too large for a match to be achieved, but friction tends to force the ball into the matched state. The friction (directed forward) reduces the spin and increases the speed of the center of mass. If the ball does not run into anything, the match of speeds is eventually made and the ball begins to roll smoothly.

If the ball is struck between the middle and the special point, its spin is in the right direction for smooth rolling but is too small. At the point of contact with the cloth the ball's surface has a small rearward motion owing to the spin and a larger forward motion owing to the speed of the center of mass. The net slipping motion there is forward, creating a rearward friction force that tends to increase the spin and decrease the motion of the center of mass until the match that causes a smooth roll is achieved.

If the ball is struck below the middle, the spin is in the wrong direction for smooth rolling. This time the friction induced by slipping reduces both the spin and the speed of the center of mass. Eventually the spin stops and the ball begins to roll smoothly.

A skilled player can impart a long or short run to a ball by striking it at a point relative to the special point for smooth rolling. If he wants the ball to reach the far side of the table quickly, he must strike it above the special point so that the friction propels the ball.

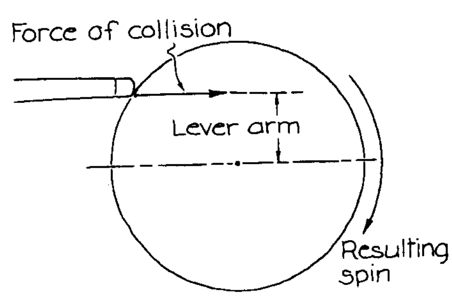
The player is more likely to be concerned with how the ball is rotating when it strikes another ball. (The other balls are called the object balls.) A collision between a cue ball and an object ball transfers momentum from the cue ball. In a head-on collision the transfer is complete, leaving the cue ball with a motionless center of mass. In a glancing collision the cue ball loses only part of its momentum and continues to travel. In any collision virtually none of the cue ball's rotation is transferred because the friction between the surfaces of the colliding balls is minute and the collision is brief. Only with significant friction could the cue ball transfer spin to an object ball.

The absence of transferred spin leads to two interesting shots. Suppose the cue ball is hit with topspin and collides head on with an object ball while it is still sliding. Just after the collision the cue ball's center of mass is motionless but the ball continues to spin. The forward-directed friction generated by the spin slows the spin and begins to propel the center of mass. Soon the cue ball begins to roll again, following the object ball. This is a follow shot. A cue ball with topspin is often said to have "follow" or "follow English."

If the cue ball is given backspin, it will return to the player after hitting an object ball head on. The collision leaves the cue ball with a motionless center of mass but with the same amount of spin. The friction generated by the sliding surface is toward the player. As the friction slows the spin and propels the center of mass, the ball begins to roll smoothly toward the player. This is a draw shot. A ball with backspin is often said to have "draw" or "draw English."

A follow shot is depicted in the middle illustration on the next page. A player who wants to pocket the four ball and the seven ball with a single shot strikes the cue ball with follow, thus causing the four ball to ricochet off the seven ball and into the pocket. The collision of the cue ball with the four ball leaves the cue ball momentarily spinning in place, but sliding friction soon propels it toward the pocket again. In the meantime the seven ball has bounced off the rail near the pocket and come into line between the cue ball and the pocket. The cue ball then pockets the seven ball and comes to a stop.

So far I have written only of stroking the cue ball along a vertical line through the center. The result is topspin or backspin about a horizontal axis. If the ball is stroked elsewhere, the axis of spin still passes through the center of mass but is no longer horizontal. A stroke on the left side of the ball is said to give left English, on the right side right English. From overhead left English is a clockwise spin about the vertical. As before, the rate of spin depends on the lever arm associated with the force. The farther



Factors in a topspin shot

off center the ball is struck, the larger the lever arm is and the faster the spin around the vertical is. The friction of the ball with the table serves only to diminish the spin.

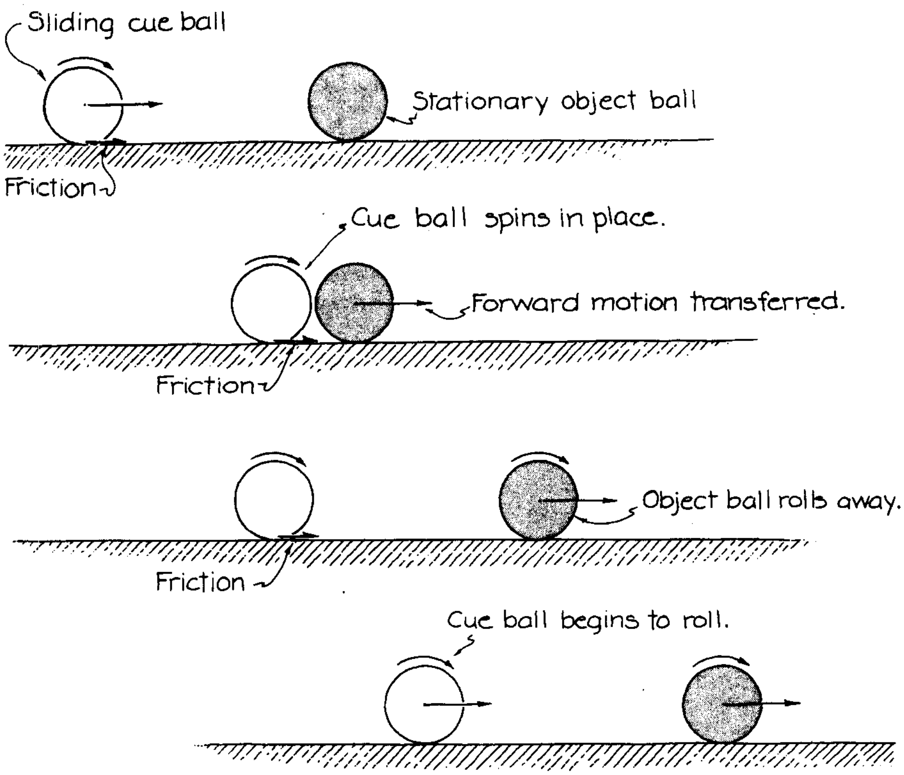
If the cue strikes the ball high or low on the side, the ball spins about an axis that is between the horizontal and the vertical. A stroke below the middle and to the left of center results in a draw with left English. The center of mass is given momentum in the usual way and the ball spins about an axis tilted out of the vertical toward the player's left. One can view this rotation as being two simultaneous spins, one that is clockwise about the vertical and another that is backspin about the horizontal. The primary friction force on the ball is imparted by the backspin.

A cue ball with side English travels in a straight line like a ball with no English, but the angle at which it rebounds from a rail is remarkably different. When the ball has no initial English, its angle of rebound is the same as its angle of approach to the rail. If the approach to the rail is perpendicular, as is shown in the bottom illustration at the left, the ball must retrace its path after rebounding. If it has left English, however, it returns along a path on the player's left because of friction during contact with the rail. From an overhead view the ball with left English turns clockwise. When it slides along the rail, the friction force is to the left. When the ball rebounds, it has not only its velocity perpendicular to the rail but also this leftward component. The ball returns along a straight path resulting from the combination of the two motions.

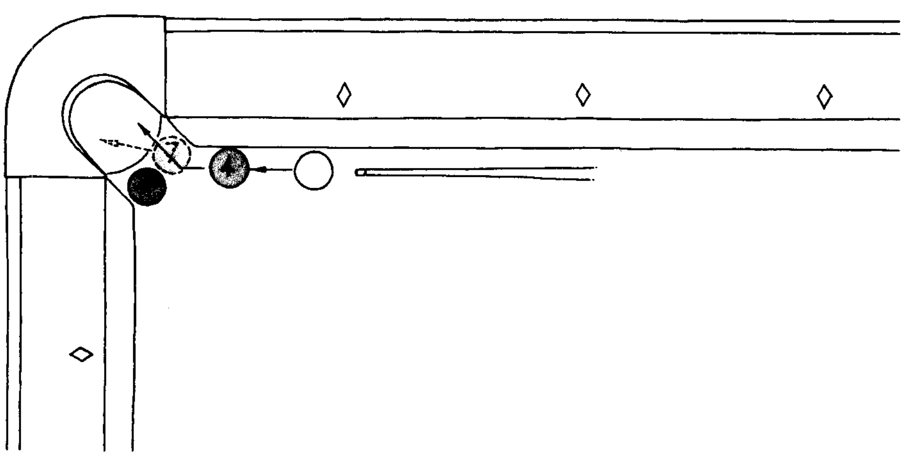
A left-English ball approaching the rail at any other angle is similarly redirected. The opposite effects arise with a right-English ball. A simple way to remember the difference is to associate the direction of English with the rotation of the rebound path: left English results in a clockwise rotation of the path and right English results in a counterclockwise rotation.

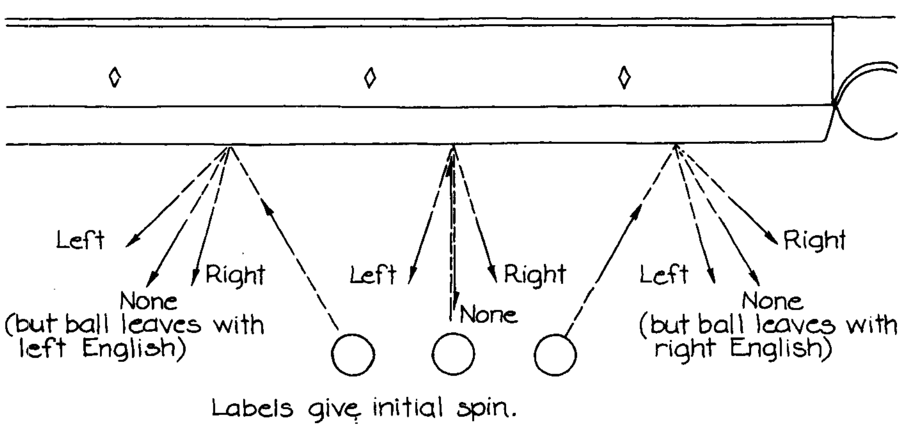
If a ball approaches the rail with no English and at some angle other than 90 degrees, friction from the rail gives it English. Consider the ball's approach to be a combination of one motion perpendicular to the rail and another parallel to it. Once contact is made the parallel component generates friction on the ball. The resulting torque spins the ball about the vertical, imparting English. Suppose a ball is sent into the rail from the player's right. If it initially had no spin, it leaves the rail with left English.

All these rotations of spin are about an axis in a plane perpendicular to the direction of travel of the ball. The massé shot supplies spin about an axis out of that plane. With the cue almost vertical the player strikes downward on the side



Four stages in a follow shot


A follow shot demonstrated



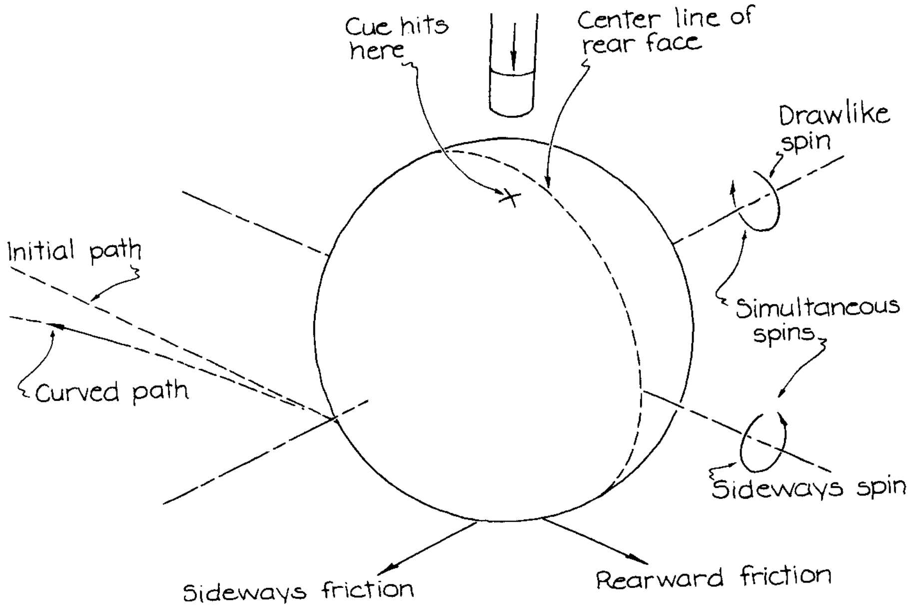
Effects of putting English on the cue ball of the cue ball. The horizontal part of the stroke determines the initial path of the ball, but the spin given to the ball generates a friction from the table that ends up curving the path.

Suppose the player strikes sharply the left side of the ball. Since the blow is hard and the lever arm is large, the spin imparted to the ball is large. The spin is about an axis that is approximately in a horizontal plane but is not perpendicular to the initial path of the ball. To simplify the spin one can regard it as consisting of two simultaneous spins about different axes. One is parallel to the initial path and the other is perpendicular to it. The spin about the axis perpendicular to the initial path is similar to that of a simple draw shot. The spin about the other axis forces the ball to slip perpendicular to the initial path, thereby generating a friction force that is also perpendicular to the path. This sideways force curves the path of the ball.

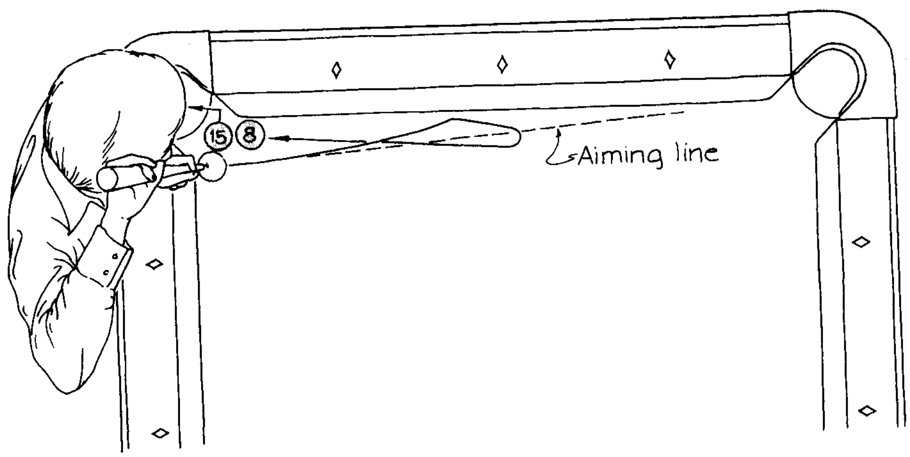
The massé is commonly employed to send the cue ball around an obstacle to reach a hidden ball. A more complex massé shot is shown in the lower illustration at the right. The idea is to sink the 15 ball and the eight ball with a single shot and to have the eight ball enter the pocket last. The cue ball is given a massé shot, knocks in the 15 ball, misses the eight ball and heads toward the rail in a curved path because of sideways friction. After rebounding from the rail the ball stops and then reverses its horizontal motion, heading back to pocket the eight ball.

The initial stroking of the cue ball gives it both backspin for a friction force like the one in the standard draw shot and sideways spin for a sideways friction to curve the path into the rail. The rebound off the rail is little affected by the backspin, but the sideways friction keeps the ball near the rail. After the rebound the backspin finally stops the horizontal motion of the ball's center of mass. Since the ball still spins, that friction force then brings it back toward the player. The remaining sideways friction continues to drive the ball toward the rail. Hence after the reversal of path resulting from the drawlike component of the shot the ball comes back near the rail to knock the eight ball into the corner pocket.

When a cue ball collides with an object ball, part of the momentum and kinetic energy of the center of mass of the cue ball is transferred. The transfers are almost total if the collision is head on. In a glancing collision the transfers cause the two balls to separate along approximately perpendicular paths. (In practice a small amount of energy is lost by the balls in the collision, and the angle between their paths is a bit less than 90 degrees. I shall disregard this complication.)



Factors in a massé shot

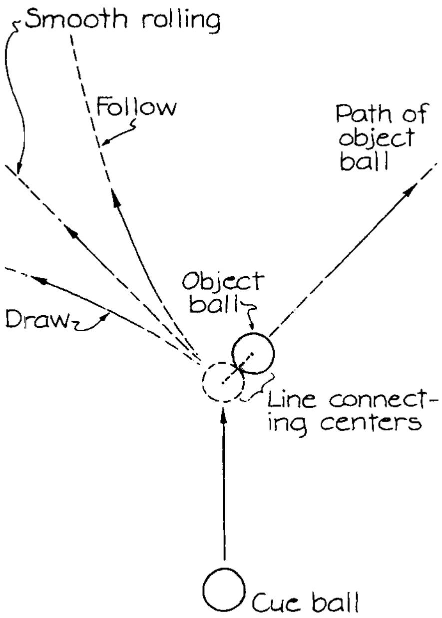


A massé shot demonstrated

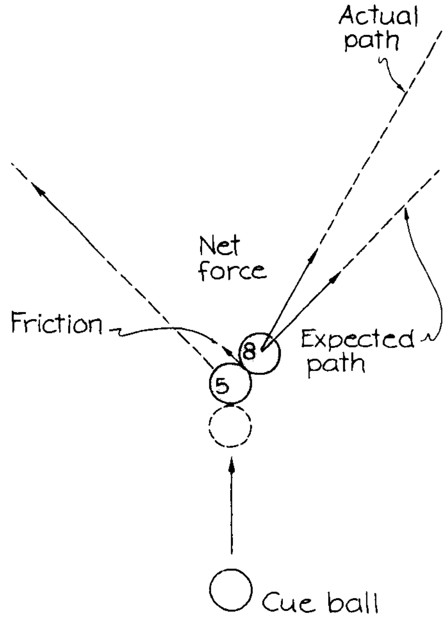
You can easily predict where the cue ball and the object ball will travel after a collision. Imagine the instant the two balls touch and mentally draw a line between their centers. At that instant the object ball acquires two forces from the cue ball. At the contact point and perpendicular to the line between the centers is a small friction force. It is almost always small enough to be ignored. Parallel to the line is a larger force that pushes the object ball off along a path that is also parallel to the line. The direction given to the object ball depends almost entirely on the orientation of the line between the centers of the balls at the instant of contact. Through experience the skilled player can direct the cue ball so that it makes contact in just the way necessary to send an object ball into a pocket. The player can be certain the cue ball will travel perpendicularly to that path.

If the cue ball is given follow or draw and still has the associated sliding when it reaches the object ball, the collision changes somewhat in that the cue ball leaves the collision site on a curved path. Suppose the cue ball has been given a large follow. The collision transfers part of the kinetic energy and momentum of the center of mass. If one can disregard entirely the friction between the colliding balls, none of the spin of the cue ball is transferred to the object ball. The cue ball begins to move away from the collision site along a path perpendicular to the path taken by the object ball. The cue ball still has topspin. The curious feature is that the ball no longer slides parallel to its path. The component of the sliding perpendicular to the path provides a sideways friction force that pushes the ball into a curved path. Therefore when the cue ball is given follow, it tends to curve back toward its original direction after a collision. A cue ball with draw tends to curve away from the original direction.

Normally the friction between two colliding balls is negligible. It can be greatly increased if the surfaces are cov-



A cue ball hits an object ball



A cue ball hits frozen object balls

ered with chalk. My favorite example comes from Byrne's book. The illustration below depicts the setup: the player must get the five ball into the pocket at the right. Can the shot be made without contact between the cue ball and the spotted ball? Normally the shot is impossible. The five ball can travel to the pocket only if the player has aligned the collision so that the line joining the centers of the cue ball and the five ball points to the pocket. The spotted ball is clearly in the way.

The shot can be made if the player chalks the left side of the five ball and then sends the cue ball into the side with a little right English. With chalk on the collision area the friction between the balls is no longer negligible. The five ball is subjected to two forces during the collision, one force parallel to the line connecting the centers and the other force (friction) perpendicular to that line. The five ball heads off in the direction of the net force, which by design is toward the corner pocket.

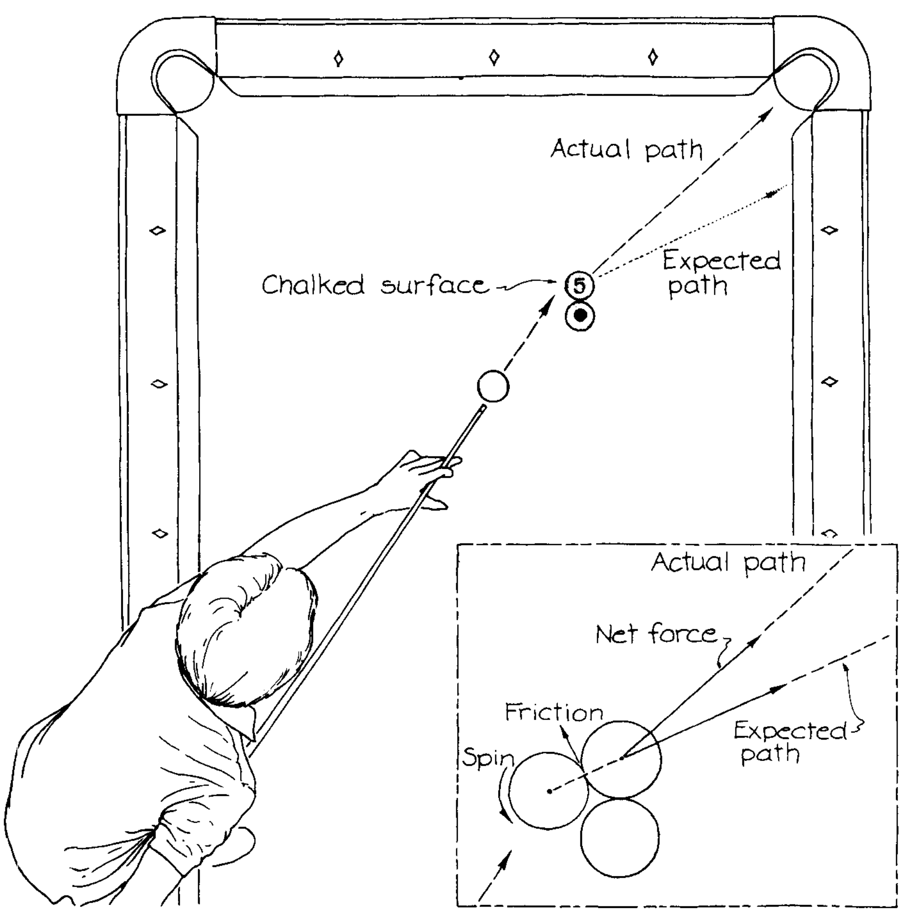
Chalking a ball is certain to get you thrown out of a pool game, but a similar application of friction between balls can be achieved more acceptably. When a cue ball strikes an object ball that is already touching another object ball (the two object balls are said to be frozen), the collision creates a friction between the object balls that can significantly alter the path of one of them. Consider the situation in the illustration at the top right on this page. The cue ball is sent directly into the five ball, which is frozen to the eight ball.

It is best to analyze the collision in two steps. First the cue ball transfers energy and momentum to the five ball, which then collides with the eight ball. The five and the eight should separate along perpendicular paths, but the eight ends up traveling more in the forward direction because of friction between the object balls.

During the collision between the five ball and the eight ball the five begins to move perpendicular to the line between the centers. The eight ball should move parallel to that line because of the force of collision from the five. Since the balls are initially frozen, however, the motion of the five ball rubs the surface of the eight ball, generating a friction force that briefly pushes the eight ball perpendicular to the anticipated path. The actual path is then set by the combination of these two forces on the eight ball during the collision; the path is more in the forward direction than it would be if the balls were not initially frozen.

My last example, the "just showin' off" shot, has become famous because Steve Mizerak, a master of pool, performed it in a television commercial. Five balls are clustered around a side pocket. The six ball lies at the mouth of a corner pocket. Can all six of the balls be pocketed with a single shot? I certainly cannot make the shot, but Mizerak is said to be successful three times out of four.

The cue ball is sent into the two ball with follow and left English. Imagine the position of the balls and the forces between them at the instant the cue ball reaches the two ball. The two ball has three forces on it. One force is parallel to the line connecting its center with the center of the cue ball. Another is along the line connecting the centers of the



Creating friction between two object balls two ball and the three ball. Because those balls were initially frozen, the two ball also has a rightward friction force on it at the point of contact when it slides off the three ball. The net force sends the two ball over to the five ball, where it ricochets into the side pocket.

Meanwhile the three ball has been moving. When the cue ball hit the two ball, the three ball received two forces from the two ball. A force parallel to the line connecting their centers knocked the three ball hard into the rail. The second force was leftward friction generated by the rubbing of the balls when the two ball departed toward the left. (This friction arises because the two ball and the three ball were initially frozen.) The rail pushes back on the three ball, directing it straight across the table. The ball also travels somewhat to the left because of its brief friction with the two ball. The three ball ends up in the pocket on the far side.

The five ball was initially frozen to the one ball and the four ball. When the two ball hits it, the five ball is subjected to several forces. One force is along the line connecting its center and the center of the two ball. Two more forces lie along lines connecting the center of the five ball with the centers of the one and the four. In addition the five ball has friction forces from being frozen to the other balls. The net force on the five ball is neatly toward the pocket in the left corner. The net force on the one ball is toward the nearby side pocket. The net force on the four ball is toward the far left pocket. Five balls are down.

After this cluster of balls has spread the cue ball returns to the area. It had been launched with follow (for a long run) and with left English. Its collision with the two ball left it traveling toward the far rail with most of its initial spin. The spin, however, is now somewhat sideways to the path. The cue ball curves to the left, rebounding closer to the corner pocket than it would without the sideways force. Its bounce from the rail removes most of the spin. Thereafter it travels in straight lines, bouncing twice more from rails. It finally reaches the six ball at the other end of the table and pockets it.

I have of course been describing only a limited selection of shots. Thousands of interesting shots remain to be analyzed. You might be particularly interested in figuring out the physics of shots into large clusters of frozen balls. Byrne has several curious examples devised by 19th-century masters of pool and billiards. You might also be interested in jump shots, in which the cue ball is sent hopping over the table or even between two tables. Be careful. Proprietors of pool halls rarely tolerate such shenanigans, even in the interest of scientific investigation.